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A Review of the Empirical Evidence on the Dimensions of Expected Stock Returns

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RESEARCH

There is a vast literature that documents the empirical evidence on the cross-section of average stock returns, and a large number of variables have been linked to expected returns [e.g., Harvey, Liu, and Zhu (2013) catalogue 315 factors for asset returns]. Since it is impossible to review every existing study, we attempt to cover topics that are related to identifying differences in the cross-section of expected returns. Campbell (2000) and Davis (2001) provide discussions of the work done in the second half of the 20th century, and this survey focuses more on the work done since then.

We start by updating some of the classic themes, such as CAPM, size, relative price, and momentum, with more recent evidence. We then move on to newer topics that have emerged in recent years, which include profitability, investment, net share issues, volatility, and liquidity. We include a brief discussion of mutual fund performance studies and conclude with some practical implications.

CAPM

The potential for empirical work in the stock market greatly increased in the 1960s due to the collection of CRSP and Compustat data[1] and the development of the Capital Asset Pricing Model (CAPM) as a theory of how expected returns should be determined. Prior to the CAPM, there was not a theoretically sound benchmark for returns.

While market beta is the only risk that should be compensated according to the CAPM, studies in the 1980s uncovered patterns in the cross-section of stock returns that contradicted this central prediction. For example, firms that have high earnings-to-price ratios (Basu, 1977, 1983), low market capitalizations (Banz, 1981), or high book-to-market equity[2] (Rosenberg, Reid, and Lanstein, 1985) were shown to be associated with high average returns, even after controlling for betas.

SIZE AND RELATIVE PRICE

In a seminal paper, Fama and French (1992) synthesized early pieces of empirical evidence against the CAPM. They examined a number of different variables in regression tests and found that firm size and book-to-market equity had the most explanatory power of the candidate variables for the cross-section of stock returns. Fama and French (1993) used this explanatory power to motivate the well-known three-factor model, which includes factors for the broad equity market, size effect, and value effect. Expanding the time period available for study, Davis, Fama, and French (2000) examined size and value premiums using data back to 1926, more than three decades earlier than the original Fama/French (1993) sample. Their results further validated the size and value effects and confirmed the common variation among stocks with similar size and book-to-market characteristics.

Rizova (2006) summarized academic research on the size effect outside of the US. The average return differences between small and large cap stocks were reliably positive in most developed markets and major emerging markets, suggesting that the size effect has been a global phenomenon. Extending the Fama and French (1998) international evidence on the value effect, Fama and French (2012) found positive value premiums in all four regions examined: North America, Europe, Japan, and Asia Pacific. They also showed that the poor, abnormal performance of small cap growth stocks (compared to a three-factor benchmark), first documented in Fama and French (1993), has persisted in the US and is present in developed markets outside the US. Rizova (2012) confirmed this finding and studied its investment implication. The results indicate that the extreme small cap growth stocks account for the bulk of abnormal underperformance of small cap growth. By excluding these stocks from a small cap strategy, Rizova (2012) showed that it is possible to better deliver the size premium while maintaining broad diversification within the small cap universe.

MOMENTUM

Jegadeesh and Titman (1993) documented the momentum effect in US stock returns: When stocks are ranked on the basis of their past three- to 12-month returns, those with the highest returns, on average, continue to outperform those with the lowest returns over the next few months—the relative performance of stocks tends to persist. In addition, the abnormal return generated by the strategy of buying past winners and selling past losers appeared to be distinct from the size and value premiums.

Momentum has also been observed in international returns. Griffin, Ji, and Martin (2003) showed that momentum profits have been economically large under both good and bad economic conditions in 40 countries. Among four developed regions (North America, Europe, Japan, and Asia Pacific), Fama and French (2012) found Japan to be the only one without return momentum.

More recently, Novy-Marx (2015) presented evidence that price momentum, rather than being an independent effect, is largely explained by earnings momentum—the tendency of stocks that recently announced strong earnings to outperform stocks that recently announced poor earnings. When recent earnings surprises are added as explanatory variables, past stock returns have no additional power to explain the variation in cross-sectional returns. In addition, momentum strategies do not retain a positive excess return

1. CRSP is the Center for Research in Security Prices at the University of Chicago. The Compustat database is produced by Standard & Poor's Corporation.

2. While book-to-market equity is widely used in academia, finance professionals may prefer using its inverse, price-to-book ratio (i.e., relative price). Here they are used interchangeably in the sense that a higher book-to-market equity is equivalent to a lower price-to-book ratio.

once controlling for market, size, value, and earnings momentum factors. The practical implication of this finding remains to be explored.

Despite the consensus on the existence of momentum, debates still exist over the cost-effectiveness of momentum strategies. There are many challenges in trying to estimate the cost of implementing any strategy historically, and this is particularly true with high turnover strategies. Since market microstructures have changed over time, trading cost estimates need to be date matched with the returns for each security at its transaction time. The estimation of trading costs, especially the implicit components such as bid-ask spread and market impact, can yield very different results depending on the model used and the assumptions made.

Lesmond, Schill, and Zhou (2004) used a price friction model to infer trading costs from daily return data. They found that the top and bottom momentum deciles tend to include stocks that are more expensive to trade, and as a result, their equally weighted long/short momentum portfolios failed to generate abnormal returns net of estimated trading costs during the sample period 1980–1998. In a study covering the period from 1967 to 1999, Korajczyk and Sadka (2004) used intra-day trade data and several models of spreads and market impact to quantify the effects of trading costs on momentum-based, long-only strategies. They showed that, depending on portfolio size and weighting scheme, estimated trading costs can either eliminate a large portion of or completely swamp the profits on momentum strategies. The break-even fund sizes—beyond which the abnormal returns are driven to zero—were estimated to be in the range from 200 million to five billion (as of 1999). The transaction costs estimated by Novy-Marx and Velikov (2014) are based on an effective bid-ask spread measure. The measure does not account for the price impact of large trades, and should thus be interpreted as the costs faced by a small liquidity demander. According to their estimation for the 1963–2012 period, transaction costs amounted to nearly half of the returns on a value-weighted, long/short momentum portfolio.

The authors also explored methods to incorporate momentum into strategy implementation in a cost effective way. In particular, they examined a technique that "screens" momentum when trading another strategy. The screened strategies, by delaying sales (purchases) and shorts (short covers) of stocks that are past winners (losers), allow investors to take advantage of momentum without increasing turnover and trading costs.

PROFITABILITY AND INVESTMENTS

One particularly fruitful development has been to explore how firms' accounting fundamentals are related to their stock returns. Some of the early work in this line of research include Frankel and Lee (1998), Cohen, Gompers, and Vuolteenaho (2002), and Titman, Wei, and Xie (2004). The main message is that expected cash flows to investors are informative of average returns.

The valuation equation,[3] which has been explored in the academic literature at least since Gordon (1959), connects the relative price and cash flow dimensions of expected stock returns. Its theoretical aspects were reviewed and synthesized by Ohlson (1990, 1995). The core of the valuation equation expresses the share price as the firm's discounted expected future cash flows to investors. That is,

$$price = sum\left(\frac{expected\ future\ cash\ flows}{discount\ factor}\right),$$

where the discount rate is roughly the long-term expected return on the stock. Dividing both sides by book equity, we have the equation expressed in terms of relative price and scaled cash flows:

$$\frac{price}{book} = sum\left(\frac{expected\ future\ cash\ flows/book}{discount\ factor}\right).$$

Building on the insights from the valuation equation, Fama and French (2006) investigated its implications for stock returns. If two stocks are expected to have the same

3. For a more formal discussion of the fundamental valuation equation, see Appendix.

cash flows scaled by book, the valuation equation implies that the one with lower relative price should have higher expected return (discount rate)—the value effect. Now suppose that two stocks are traded at the same relative price but one has higher expected cash flows, then it must also have higher expected return for the equation to hold. More specifically, since profits increase cash flow, we should see a positive relation between profitability and expected returns in the cross-section after controlling for relative price; likewise, as investments tend to decrease cash flow to shareholders, when accounting for relative price, there should be a negative investment effect across stocks.

To test these predictions, an empirical challenge has been to identify proxies that are informative about expected profitability and investment—unlike size and price-to-book, expected future cash flows are not directly observable. The proxies proposed in the literature include earnings-to-book and total asset growth (Fama and French, 2006, 2008a), and return-on-assets and investment-to-assets (Chen and Zhang, 2010). More proxies for profitability include gross profits-to-assets by Novy-Marx (2013) and profitability[4] by O'Reilly and Rizova (2013). Using these proxies, studies generally confirmed the predictions of the valuation equation. Controlling for relative price, investment and profitability have additional explanatory power for the cross-section of expected returns. Average returns are positively correlated with profitability and inversely related to investment.

Evidence on cash flow dimensions, backed by the valuation equation, suggests that there is potential for improving existing asset pricing models. Fama and French (2014, 2015a) showed that the five-factor model that includes profitability and investment factors[5] provides a more complete description of the cross-section of expected stock returns than the original three-factor model. Compared with existing models, it better captures the extremely low average returns of small growth stocks. The additional factors also help to improve the explanatory power for some other return patterns discussed below, including those related to volatility and net share issues.

NET SHARE ISSUES

A number of studies examined the link between public share issuance activities and long-run stock average returns: Ikenberry, Lakonishok, and Vermaelen (1995) documented that firms tend to have positive abnormal returns during the four years after an open market share repurchase announcement. Loughran and Ritter (1995) found that companies that issued stock, either through IPO or seasoned equity offering, have generated poor returns in the five years following the issue compared to non-issuers. Through a reexamination of long-term return anomalies, Fama (1998) showed that the above abnormal returns associated with share issuance have largely disappeared after controlling for size and book-to-market effects.

Motivated by the observations in these long-term event studies, researchers investigated whether share issuance conveys information about cross-sectional differences in returns. Unlike earlier studies that rely on public announcements of stock issues and repurchases, Daniel and Titman (2006) measure net share issuance using the change over the past five years in shares outstanding—an approach that allows them to cover all issues and repurchases including those not publicized. Focusing on shorter investment horizons, they showed that net share issuance appears to be negatively related to subsequent average monthly returns in the cross-section. Pontiff and Woodgate (2008) used a similar annual issuance measure in their regression analysis and confirmed that there has been a reliable relation for holding periods ranging from one month to three years in post-1970 data.[6] Fama and French (2014) studied portfolios formed by sorting stocks into net share issues groups. While average returns were flat across the lowest three quintiles of positive net share issues, they did appear to be much lower in the highest quintile.

Fama and French (2008b) suggested that the explanatory power of net share issues can be attributed to the information therein about expected cash flows. Since firms that issue stock tend to have high investment relative to earnings, and the opposite holds for firms that repurchase stock (Fama and French, 2005), net share issues provide

4. Operating income before depreciation and amortization minus interest expense scaled by book equity.

5. Their proxy for expected profitability is similar to the one used in O'Reilly and Rizova (2013), and their proxy for expected investment follows from Fama and French (2006).

6. They also note that neither issuance measures work in pre-1970 data because share issuance is drastically different before and after 1970.

information about expected cash flows to help better estimate expected returns across stocks. Consistent with this intuition, Fama and French (2014) showed that the returns of portfolios formed on net share issues can be well explained by the five-factor model that includes profitability and investment factors. In other words, the net share issues anomaly is subsumed by known drivers of expected returns.

BETA AND VOLATILITY

Since the earliest tests of the CAPM, researchers have shown that the empirical security market line, which depicts the relation between return and risk, is too flat. For example, Black, Jensen, and Scholes (1972) and Vasicek and McQuown (1972) reported that low beta stocks have been associated with positive CAPM alphas. More recently, there has been a surge in academic interest in the performance of low market beta and low volatility stocks.[7]

By rebalancing the volatility deciles monthly and equally weighting the stocks in each decile, Blitz and van Vliet (2007) found a positive CAPM alpha spread between the bottom and top volatility deciles, which cannot be fully explained by size, value, and momentum effects. This phenomenon was observed in both the global and regional (US, Europe, and Japan) stock markets. Baker, Bradley, and Wurgler (2011) confirmed the volatility effect in the US: The highest-beta (volatility) quintile portfolios, where stocks are market cap-weighted and rebalanced monthly, have underperformed their low counterparts in terms of both raw returns and CAPM alphas. The long/short volatility portfolios proposed by Frazzini and Pedersen (2014)[8] use monthly rebalancing and an unconventional weighting scheme—stocks are held in proportion to their beta or volatility ranking. The portfolios were shown to produce significant excess returns in the US and global equity, Treasury, credit, and futures markets, accounting for their exposure to market, value, size, momentum, and liquidity factors. Crill (2014) presented the results for volatility and market beta quintiles that are annually rebalanced and market cap-weighted. The quintiles' returns are well explained by their size, value, and momentum loadings, which implies that the results in previous studies may be magnified by their rebalancing frequency and/or weighting scheme. While using an extreme weighting or rebalancing scheme may give rise to a "low volatility anomaly", the relevance of this observation to a well-diversified, cost-effective, real-world strategy is less obvious.

Fama and French (2014) and Novy-Marx (2014)[9] showed that, except for the extremely low returns to the highest-beta (volatility) stocks, market cap-weighted beta (volatility) portfolios have yielded similar returns over the past decades. More importantly, they found that the return patterns can be well explained by known drivers of expected returns such as relative price, profitability, and investment, some of which were not accounted for in previous studies. In particular, the returns of low beta (volatility) stocks behaved like stocks of profitable firms with low levels of investment trading at low relative prices. It is unclear, however, whether low beta (volatility) strategies will maintain their emphasis on these premiums going forward. Crill (2014) provided evidence that the emphasis on the relative price premium has been inconsistent and mostly isolated to the past few decades—low volatility portfolios had no such tendency prior to the sample periods studied in Fama and French (2014) and Novy-Marx (2014). Therefore, although low volatility strategies have achieved appealing market-like returns with lower than market volatility over the past 40 to 50 years, caution is warranted in expecting that they will continue to do so.

RESIDUAL VOLATILITY

A related line of research focuses on residual volatility[10]—the volatility that is left after subtracting systematic volatility due to common sources of return variation. While financial theory suggests residual volatility should not be related to expected returns because it can be diversified away, there has been empirical evidence that supports the opposite. In both US and other developed markets, Ang et al. (2006, 2009) found that low residual volatility stocks have outperformed high residual volatility stocks, where residual volatility is measured by the standard deviation of past

7. Here beta and volatility are grouped together because they tend to be highly correlated. While some studies use past volatility to sort stocks and some use past market betas, both metrics tend to yield similar results.

8. They also hypothesize that many investors are constrained in the leverage they can take, so they overweight risky high-beta securities; this leads to lower risk-adjusted returns required by high-beta assets than low-beta assets, which require leverage.

9. Novy-Marx (2014) also documented that sorting on past volatility produced a similar but more pronounced volatility effect than sorting on past market beta. For example, the underperformance of the top volatility quintile was greater than that of the top beta quintile.

10. Academic papers often refer to residual volatility as idiosyncratic volatility.

month residuals relative to the Fama/French three-factor model. In particular, stocks with high residual volatility have had abysmally low average returns that cannot be explained by exposure to aggregate volatility risk nor size, relative price, momentum, and liquidity effects.

Bali and Cakici (2008), however, suggested that the results may be sample and methodology specific. The negative relation between residual volatility and the cross-section of expected returns either weakened or disappeared when they tested it using a different data set, return frequency, or portfolio specification. Other studies proposed that the residual volatility effect is an artifact of one-month return reversals (Fu, 2009; Huang et al., 2009) and measurement biases due to market microstructure (Han and Lesmond, 2011).

Crill and Shah (2012) showed that, despite the attention it has generated, residual volatility may not be useful to improve real-world investment strategies. The relation between residual volatility and average returns is concentrated among micro and small cap stocks and mostly driven by the poor performance of high residual volatility stocks, which only represent a small segment of the market.[11] The return spread is also substantially reduced after controlling for small growth and downward momentum, so incorporating residual volatility is unlikely to improve strategies that already monitor these effects.

LIQUIDITY

Amihud (2002) presented evidence that expected returns are inversely proportional to liquidity, which suggests that investors demand a premium for holding less liquid stocks. In this study, illiquidity is proxied by the price impact of trade volume, as illiquid stocks tend to experience greater price changes in response to the temporary price pressure created by large order flows than liquid stocks. Pástor and Stambaugh (2003) proposed an alternative proxy for liquidity risk by measuring liquidity beta—the sensitivity of stock returns to marketwide liquidity shocks. Controlling for size, value, and momentum factors, they found that, on average, stocks with high liquidity betas have outperformed stocks with low sensitivities. Other studies employed turnover-based liquidity measures. For example, Haugen and Baker (1996) used the ratio of annual average trading volume to market capitalization and its trend to define low vs. high liquidity (turnover) stocks.

Crill, Davis, and Lee (2014) revisited the return-liquidity relation by analyzing various commonly used metrics of liquidity, including the aforementioned liquidity and liquidity risk proxies. Their analysis suggested that return spreads have not been robust across different time periods, firm sizes, and different proxies for liquidity. While the lack of a reliable liquidity premium may be specific to the sample period under review, or due to the proxies not measuring illiquidity properly, there is also a possibility that liquidity and liquidity risk are not meaningful drivers of expected returns. The authors pointed out that, regardless of the interpretation, it is still important to consider the impact of illiquidity on trading costs when managing portfolios. Using a flexible and patient trading approach is likely to result in more favorable transaction prices and lower costs, compared to liquidity seekers who demand immediacy when trading a security.

MUTUAL FUND PERFORMANCE

Methods for analyzing mutual fund performance have evolved with advances in empirical asset pricing research. Since the Fama/French three-factor model largely replaced CAPM for return-adjustment in empirical studies, it has also become a standard benchmark to measure mutual fund excess returns, or alpha. More generally, one can utilize empirical factor models to evaluate a fund's performance in excess of its exposure on the known drivers of expected returns—typically company size and relative price, but also momentum and profitability more recently. With this evaluation tool, researchers have attempted to answer interesting questions, such as whether portfolio managers possess sufficient skill to consistently generate positive alphas.

Carhart (1997) tested for persistence in fund returns using a sample of 1,892 equity funds from 1962 to 1993. He found

11. In their sample the highest 20% of residual volatility stocks represent 2.4% of the total market capitalization.

little evidence that managers can consistently outguess the market—past winners did not generate positive abnormal returns in the subsequent period after controlling for size, relative price, and momentum. To disentangle luck from skill, Fama and French (2010) used bootstrap simulations[12] to build a distribution of manager outcomes assuming no alpha and compared that with the actual performance of 3,156 active US equity mutual funds from 1984 to 2006. It turns out that the observed outperformance is no more than what would be expected just by chance, and overall, very few funds seem to have sufficient skill to cover costs. Barras, Scaillet, and Wermers (2010) developed a technique to correct for luck in realized alphas and estimate the true proportion of zero-, positive-, and negative-alpha funds. When performance is measured over the whole sample period (1975–2006), they found that only 0.6% out of 2,076 equity funds has true positive alpha and the percentage of negative-alpha funds is much higher at 24%. The findings are not confined to US equity—similar conclusions can be found in studies of UK equity mutual funds (Quigley and Sinquefield, 2000; Cuthbertson, Nitzsche, and O'Sullivan, 2008), international and global equity mutual funds (Breloer, Scholz, and Wilkens, 2014) as well as bond mutual funds (Lee, 2009). One interpretation of these results is that market prices are fair and difficult to outguess. These studies highlight the importance for managers to have a deep understanding of the systematic drivers of differences in expected returns among securities and minimize unnecessary costs.

Compared with mutual funds, hedge funds are more challenging to evaluate. Due to their less constrained investment mandate, fee structures, and more discretion in how/when returns are historically reported to hedge fund databases, the returns of hedge funds have been more challenging for standard factors to explain. Respecting the reporting of hedge fund returns, the data often suffers from several limitations, including self-selected reporting, return managing, survivorship bias, and stale pricing of illiquid assets. Based on different methodologies to benchmark returns and correct for data biases, studies have reported mixed results regarding hedge fund skills and performance persistence (see, for example, Malkiel and Saha, 2005; Kosowski, Naik, and Teo, 2007; Ibbotson, Chen, and Zhu, 2011). To encompass the issues of returns data, Griffin and Xu (2009) focused on holdings data from funds' required filings of their long-equity positions. They found no evidence that hedge funds are better at long-equity investment than mutual funds.

CONCLUSIONS

To explain the cross-section of expected stock returns, a large number of candidate variables have been examined in recent decades. The findings of this body of research have advanced our knowledge of financial markets, and as a result, our investment solutions have also evolved. In this process, however, it is important to exercise great caution when interpreting and applying empirical research. Before following a pattern found in historical data, investors should be mindful of data-mined results. As noted by Harvey, Liu, and Zhu (2013), given the extensive search, many variables would be deemed "significant" by chance. Although there is no guarantee that any premium will be positive over an investment lifetime, we have greater confidence in the premiums that are sensible and hold up across different sample periods, markets, and variable specifications.

Premiums should be evaluated not only in isolation but also along with other premiums. The reason is that, as shown in Fama and French (2015b), when a variable is added to a set of existing variables, its incremental power to produce return spreads is typically less than when examined alone. In extreme cases, the effect associated with one variable can be subsumed by others—recall a few reviewed in this survey. When designing portfolios, it is then crucial to consider the interactions between premiums and target the dimensions that do expand the investment opportunity set.

Besides the science of identifying sources of higher expected returns, the implementation aspect plays an equally important role. To qualify as the foundation of an investment strategy, a premium should lead to well-diversified and cost-effective portfolios. If portfolio turnover must be very high to capture the premiums, there may be little advantage to investors after implementation costs. If the relation between return and a sorting variable is concentrated in a tiny segment

12. A bootstrap simulation is a method of analysis that can be used to approximate the probability of certain outcomes by running multiple trial runs, called bootstrapped samples, using historical returns. The projections or other information generated by bootstrapped samples regarding the likelihood of various investment outcomes are hypothetical in nature, do not reflect actual investment results, and are not guarantees of future results. Results will vary with each use and over time.

of the market—and flat or even opposite elsewhere—it becomes more challenging to construct portfolios that achieve diversification and continuous exposure to the premiums. Since most studies do not address these issues directly, it is important to understand the difference they can make between simulated numbers and those that can be achieved in real-world portfolios.

Based on the above considerations, it is not surprising that only a few variables have survived the scrutiny and become consistent dimensions of expected returns. At Dimensional, we have built equity strategies along those dimensions—size, relative price, and profitability. By continuously balancing the tradeoffs between competing premiums, diversification, and costs, our strategies are well positioned to capture the premiums on an after-cost basis. In the meantime, as we keep on refining and innovating investment solutions, rigorous empirical research will continue to be the foundation of this process.

APPENDIX

This appendix provides a more formal discussion of the fundamental valuation equation that underlies the relative price and cash flow dimensions of stock expected returns. The notation follows Fama and French (2006).

According to the dividend discount model of stock prices, the market price of equity equals the sum of the present values of expected future dividend payments:

$$M_t = \sum_{\tau=1}^{\infty} \frac{E(D_{t+\tau})}{(1+r)^{\tau}}$$

where M_t is the stock's price at time t and $E(D_{t+\tau})$ is the expected dividend at future time $t+\tau$. Since the dividend is paid at a future time, it should be discounted back to time t. The rate at which it should be discounted is r, roughly the long-term expected return on the stock.

The earnings of a firm are either paid out to investors as dividends or retained for investment. Assuming that clean surplus accounting holds, the dividend D_t can be expressed as the equity earnings Y_t minus the change in book equity $dB_t = B_t - B_{t-1}$. This implies the market value of equity (divided by book equity) is

$$\frac{M_t}{B_t} = \sum_{\tau=1}^{\infty} \frac{E(Y_{t+\tau} - dB_{t+\tau})/(1+r)^{\tau}}{B_t}.$$

First note that the price-to-book shows up on the left-hand side of the equation. If we control for the cash flow quantities (Y and dB) relative to current book equity, then a lower price-to-book ratio implies a higher expected return r—the value effect. Second, controlling for relative price and investment, the expected return should be positively related to expected earnings, hence a positive profitability effect. Finally, fixing price-to-book and profitability reveals a negative investment effect: Firms with higher expected investment should generate lower expected returns.

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